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Perturbation Analysis of Transonic Wind Tunnel Wall Interference

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The wind tunnel wall interference at transonic speeds is considered as a perturbation to the basic flow around the airfoil in free air. Based on the transonic small disturbance theory, the perturbation equation is derived from the nonlinear transonic equation and is linear but with variable coefficients containing the nonlinear solution of the basic flow. With the boundary conditions imposed on the tunnel wall, the equation is solved numerically by a direct matrix method. The solutions agree well with those directly calculated from the small disturbance equation. The present method is convenient to use for practical wall interference calculations as only a linear equation is solved. Based on the present results, the applicability of the subsonic linear interference theory in the transonic range is discussed.

Nomenclature

c	= chord of airfoil
C_L	= lift coefficient
C_p	= pressure coefficient
F	= thickness distribution of airfoil
H	= half height of wind tunnel
$\bar{H}$	= half height of wind tunnel in transonic similarity coordinate
K	= transonic similarity parameter
M_∞	= free stream Mach number
P	= porosity factor
x,y	= transonic similarity coordinates
α	= angle of attack
δ	= thickness of airfoil
γ	= ratio of specific heats
ϕ	= disturbance potential

Subscripts

0	= free airflow
I	= interference flow

Introduction

THERE are two existing methods of calculating the interference of wind tunnel walls in the flow around a tested model in transonic flows. In the first method, the flow bounded by the tunnel walls is computed directly from the nonlinear transonic equations.[1,2] The second method argues that the farfields of the subsonic and the transonic flows are highly similar, thus the subsonic linear theory of wall interference can be applied in the transonic range.[3] The latter method has been shown to work reasonably well and requires substantially less computation effort.[3,4] However, in transonic flows the flow around the airfoil is definitely nonlinear and, as the subsonic linear solution is being applied to the nearfield, the error that it may induce is yet unknown. This note presents a method of calculation of the interference flow as a perturbation to the nonlinear free airflow. The governing equation of the interference flow is a perturbation equation derived from the transonic small disturbance equation. The equation is linear and is solved by satisfying the imposed conditions at the tunnel wall. By checking against the direct nonlinear calculations the method is shown to be accurate and thus can be used for practical calculations of interference corrections. Since it is a perturbation method, the results can be compared directly with those of the subsonic linear theory and consequently provide assessment of the applicability of the latter method in the transonic range.

Analysis

The airfoil is situated in the middle of the wind tunnel with height $2H$. The tunnel walls are perforated for flow ventilation in transonic tests. The restriction induced by the tunnel wall is regarded as a small perturbation to the basic free airflow. The interference potential ϕ_I has the order of magnitude of $1/H$ and is assumed to be one order higher than the free air potential ϕ_o. Within the framework of the transonic small disturbance theory,[1,5] the governing equations and the boundary conditions for the free air and the interference flows can be derived as follows.

Free airflow:

$$[K-(\gamma+1)\phi_{ox}]\phi_{oxx}+\phi_{oyy}=0 \tag{1}$$

$$(\phi_{oy})=F_x-\frac{\alpha}{\delta} \qquad \text{at } y=\pm 0 \text{ on airfoil} \tag{2a}$$

$$\phi_{ox},\phi_{oy}\to 0 \qquad \text{as } x^2+y^2\to\infty \tag{2b}$$

The shock condition states:

$$\langle K+(\gamma+1)\phi_{ox}\rangle[\phi_{ox}]^2+[\phi_{oy}]^2=0 \tag{2c}$$

$$[\phi_o]=0 \tag{2d}$$

Interference flow:

$$[K-(\gamma+1)\phi_{ox}]\phi_{Ixx}-(\gamma+1)\phi_{oxx}\phi_{Ix}+\phi_{Iyy}=0 \tag{3}$$

$$\phi_{Iy}=0 \qquad \text{at } y=\pm 0 \text{ on airfoil} \tag{4a}$$

$$\phi_{Ix}\pm\frac{1}{P}\phi_{Iy}=-\left(\phi_{ox}\pm\frac{1}{P}\phi_{oy}\right) \text{ at } y=\pm\bar{H} \tag{4b}$$

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$$< (\gamma+1)\,[\phi_{1x}(x_o,y)+\phi_{oxx}(x_o,y)x_1{}^D(y)\,] > = 2x_{oy}{}^D x_{1y}{}^D \tag{4c}$$

$$[\phi_1(x_o,y)+\phi_{ox}(x_o,y)x_1{}^D(y)\,]=0 \tag{4d}$$

where $\tilde{H}=H\delta^{1/3}M_\infty^{1/2}$, the tunnel half height in the transonic similarity coordinate. The shock condition for the perturbation equation has been transferred to the zero order shock location by analytic continuation.[6] The final shock location is thus

$$x^D = x_o^D(y) + x_1^D(y) \tag{5}$$

The zero order equation and the boundary condition can be identified with the transonic small disturbance flow for a two-dimensional airfoil in free air.[5] The method of solution is well documented and computer codes are available.[7] The first-order equation is linear with the coefficients containing the zero-order solutions, and can be written in a finite difference form in a similar manner as the zero-order equation. The finite difference equations are then solved directly by an existing method for the solution of a system of linear algebraic equations. Details of the numerical procedure for the solution of the interference equation are given in Ref. 8.

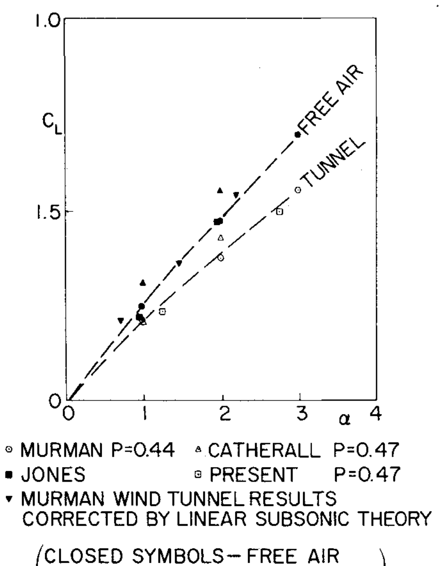

Fig. 1 Comparison of different results for C_L vs α for NACA 0012 airfoil in wind tunnel with perforated walls and free air at $M_\infty = 0.80$, $H/c = 2$.

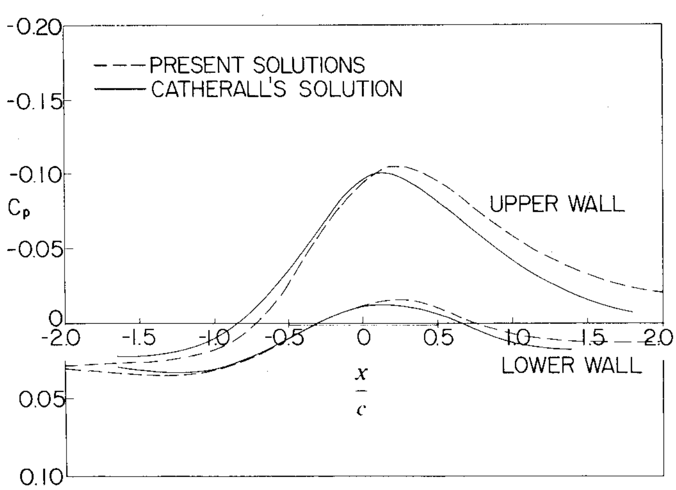

Fig. 2 Comparison of pressure distributions at wind tunnel walls for NACA 0012 airfoil at $M_\infty = 0.8$, $H/c = 2$, $P = 0.47$, and $\alpha = 1$ deg.

In the classical method of tunnel wall interference, the interference potential is calculated without enforcing the boundary condition on the airfoil surface by considering that the airfoil shrinks to a point singularity. The corrections are obtained by evaluating the velocity components of the interference flow at the location of the singularity. This approach is justified if the tunnel height is much larger than the chord of the airfoil. This condition is compatible with the present perturbation assumption, thus the classical approach is adopted for the evaluation of corrections for the interference flow.

Results and Discussion

In order to establish the accuracy of the method, a case was calculated and the results tested against those obtained by direct solutions of the transonic small disturbance equation for wind tunnel flows. The tested case chosen has the conditions: NACA 0012 airfoil, $M_\infty = 0.8$, $H/c = 2$, and $P = 0.44$ and 0.47, it was calculated by Murman[1] and Catherall.[9] Details of tunnel wall pressure distributions are also available for Catherall's solution.[10]

The results for lift vs angle of attack are shown in Fig. 1. The free air solutions for the present calculation are obtained from Jone's code[7] and are in good agreement with Murman's results. The values for the wind tunnel are computed by adding the first-order "corrections" to the free air solutions. The results of the first-order solution agree reasonably well with those of Murman's direct solutions. Catherall's solutions, for both the free air and the wind tunnel, are higher than the present and Murman's results. The pressure distributions on the tunnel walls are shown in Fig. 2 with Catherall's direct solutions for comparison. The present results follow closely those of Catherall. Thus, the present perturbation method provides an accurate approximated solution to the fully nonlinear problem.

The method is now adopted to assess the accuracy of the subsonic linear theory in the transonic range. The test case has the following specifications: 16% thick supercritical airfoil, $M_\infty = 0.6$, $H/c = 3$, $P_{\text{upper}} = 1.5$, $P_{\text{lower}} = 0.5$. This case is chosen because its wind tunnel test data include the tunnel wall pressure measurements, which are crucial for a realistic determination of the interference flow inside the tunnel,[8] and, in addition, the tested range covers both subcritical and supercritical flows. The porosity factors were derived from the measured wall pressure distributions by the method given in Ref. 3. The corrections for angle of attack and freestream Mach number for this case are shown in Fig. 3. At lower lift, the flowfield is completely subsonic, the results of the sub-

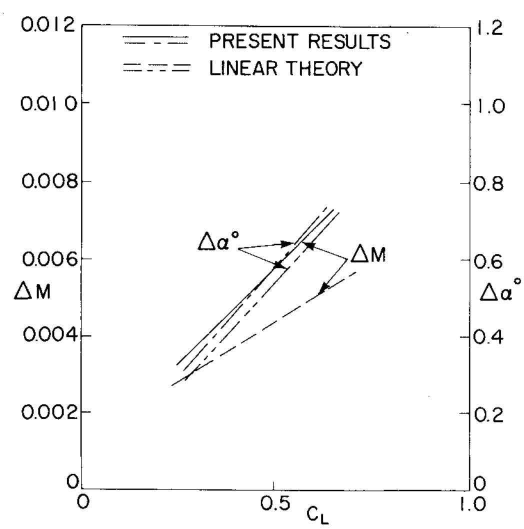

Fig. 3 Corrections for angle of attack and freestream Mach number from present method and subsonic linear theory.

sonic linear theory are fairly close to the present ones. At higher lift, where the flow over the upper surface of the airfoil is supercritical, the present results give a slightly higher angle of attack correction and much greater value in Mach number correction than those of the subsonic linear theory. Thus, for angle of attack correction, the subsonic linear theory can be applied with reasonable accuracy in the transonic range. This has also been shown by Murman[1] who demonstrated that the wind tunnel results, corrected by subsonic linear theory, follow very closely the free air results calculated directly from the nonlinear equations. These corrected results (Murman's) are also shown in Fig. 1. The low value of Mach number correction of the subsonic linear theory has also been reported by Blackwell in the study of the blockage corrections.[11] One may observe that by examining the farfield asymptotic solution of the transonic equation,[1,5] the circulatory flow is similar to that of the subsonic flow, but the nonlinear terms appear as an additional doublet. Thus the displacement flow will be affected by the nonlinear compressibility. This is particularly significant when the wall is located well within the nearfield. For this case, the lateral distance of the wall from the airfoil, in the transonic similarity coordinate $H\delta^{1/3}M_\infty^{1/2}$, is only 1.27, while the asymptotic solution applied at $|y| \geq 6$. The subsonic linear theory can be considered as a farfield method and has a close similarity in concept to the farfield matching method in the transonic flow computations.[12] It has been demonstrated that for a lateral distance of $|y| < 3$ the first-order farfield solution based upon the Prandtl-Glauert equation is insufficient and the second-order solution which accounts for the nonlinearity of the flow should also be included. For engineering application, however, the error of Mach number correction by the subsonic linear theory may be well within the accuracy of the measurements.

In summary, the transonic perturbation formulation as presented provides a consistent method for the transonic wind tunnel wall corrections. The tunnel wall pressure measurements can also be used as input, thus avoiding the uncertainty of determining the porosity factor of the tunnel walls. The subsonic linear theory produces a reasonable approximation in the transonic range and is simple to use for practical applications.

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